

first half page of nuclides chart (symbol ϵ stands for electron capture/ β^+ decay)

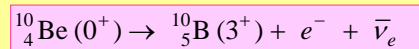
→ 27 isotopes: 8 β^- decays, 6 β^+ decays, spanning 16 orders of magnitude in rate!

Abundance or Half-life					Abundance or Half-life					
Z	A	Atomic mass (u)	I ^π		Z	A	Atomic mass (u)	I ^π		
H	1	1.007825	$\frac{1}{2}^{+}$	99.985%	10	10.012937	3^{+}		19.8%	
	2	2.014102	1^{+}	0.015%	11	11.009305	$\frac{3}{2}^{-}$		80.2%	
	3	3.016049	$\frac{1}{2}^{+}$	12.3 y (β [−])	12	12.014353	1^{+}		20.4 ms (β [−])	
He	2	3.016029	$\frac{1}{2}^{+}$	$1.38 \times 10^{-4}\%$	13	13.017780	$\frac{3}{2}^{-}$		17.4 ms (β [−])	
	4	4.002603	0 ⁺	99.99986%	C	6	9.031039	$\frac{3}{2}^{-}$		0.13 s (ε)
Li	3	6.015121	1^{+}	7.5%		10	10.016856	0 ⁺		19.2 s (ε)
	7	7.016003	$\frac{3}{2}^{-}$	92.5%		11	11.011433	$\frac{3}{2}^{-}$		20.4 m (ε)
	8	8.022486	2^{+}	0.84 s (β [−])		12	12.000000	0 ⁺		98.89%
Be	4	7.016928	$\frac{3}{2}^{-}$	53.3 d (ε)		13	13.003355	$\frac{1}{2}^{-}$		1.11%
	8	8.005305	0 ⁺	0.07 fs (α)	14	14.003242	0 ⁺		5730 y (β [−])	
	9	9.012182	$\frac{3}{2}^{-}$	100% slowest	15	15.010599	$\frac{1}{2}^{+}$		2.45 s (β [−])	
	10	10.013534	0 ⁺	1.6 My (β [−])	N	7	12.018613	1^{+}		11 ms (ε)
	11	11.021658	$\frac{1}{2}^{+}$	13.8 s (β [−])		13	13.005739	$\frac{1}{2}^{-}$		9.96 m (ε)
B	5	8.024606	2^{+}	0.77 s (ε)		14	14.003074	1^{+}		99.63%
	9	9.013329	$\frac{3}{2}^{-}$	0.85 as (α)	15	15.000109	$\frac{1}{2}^{-}$		0.366%	
					16	16.006100	2^{-}		7.13 s (β [−])	

Some anomalies:

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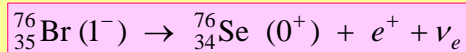
1. According to our theory, the very slow decay: (1.6×10^6 yrs)



should not occur at all, because angular momentum does not add up, i.e.:

$$\vec{0} \neq \vec{3} + (\vec{0} \text{ or } \vec{1})$$

2. Another example: (16.1 hr)



This should not occur because the wavefunctions in the nuclear matrix element have opposite parity, so the integrand is odd and should vanish:

$$M_{\text{nuclear}} \equiv \int \psi_f^*(\vec{r}) \psi_i(\vec{r}) d^3r = 0 \quad ???$$

Forbidden Decays:

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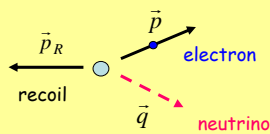
These are two examples of **forbidden decays** - they cannot proceed under the allowed approximation, since

$$M_{if} = G \int \psi_{p,f}^*(\vec{r}) \phi_e^*(\vec{r}) \phi_\nu^*(\vec{r}) \psi_{n,i}(\vec{r}) d^3r = 0 \text{ if } \phi_e^*(\vec{r}) \phi_\nu^*(\vec{r}) = \frac{1}{V}$$

Is there some other way they can occur?

Reconsider the electron and antineutrino wave function as a **multipole expansion**:

$$V \phi_e^*(\vec{r}) \phi_\nu^*(\vec{r}) = e^{i\vec{p}_R \cdot \vec{r} / \hbar} \equiv \sum_{L=0}^{\infty} i^L (2L+1) j_L(p_R r / \hbar) P_L(\cos \theta)$$



j_L = spherical Bessel Function
 $P_L(\cos \theta)$ = Legendre polynomial

Beyond the "allowed approximation":

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$$e^{i\vec{p}_R \cdot \vec{r} / \hbar} \equiv \sum_{L=0}^{\infty} i^L (2L+1) j_L(p_R r / \hbar) P_L(\cos \theta)$$

spherical Bessel functions:

$$j_0(x) = \frac{\sin x}{x}; \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \dots$$

with $x = p_R r / \hbar$

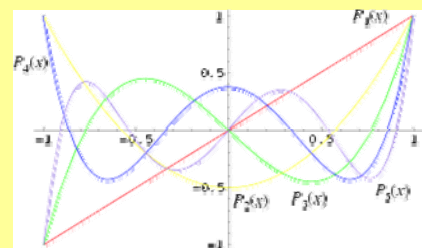
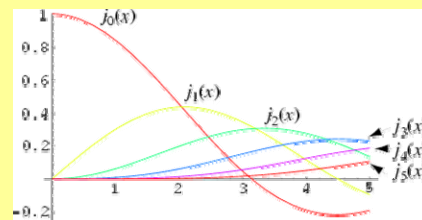
for successively larger L, they become more significant for larger recoil momentum
 → this will change the momentum dependence of our prediction!

Legendre polynomials:

$$P_0 = 1, \quad P_1 = x, \quad P_2 = \frac{1}{2}(3x^2 - 1) \dots$$

with $x = \cos \theta$

these introduce a new angular dependence to the integrand for M_{if} → equivalent to angular momentum L



- angular momentum coupling for the multipole order L , together with S and nuclear angular momentum allows previously impossible reactions to proceed
- multipole term has parity $(-1)^L$, which allows nuclear states of opposite parity to be "connected" by the beta decay operator
- momentum dependence of the matrix element varies as $(p_R r / \hbar)^L$...

since this is small, the lowest multipole order L that satisfies the conservation laws will dominate the transition

$$\text{rate} \sim |M|^2 \sim (p_R r / \hbar)^{2L} \equiv (0.01)^{2L} \rightarrow \text{dramatically smaller for large } L$$

momentum dependence also affects the shape of the spectrum; Kurie plots are not linear unless "shape factors" are taken into account....

- naming convention:

$L = 0$	allowed
$L = 1$	first forbidden
$L = 2$	second forbidden
$L = 3$	third forbidden....

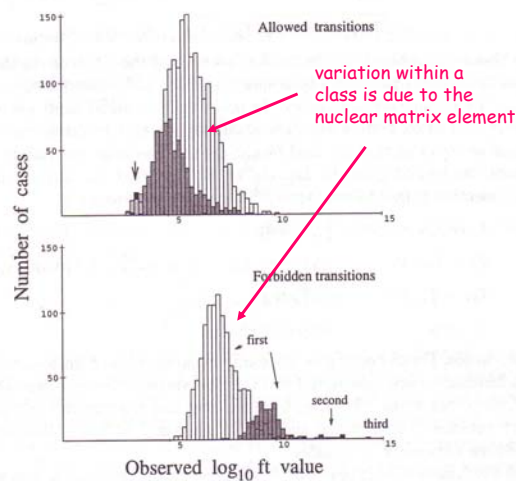
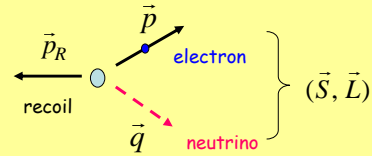
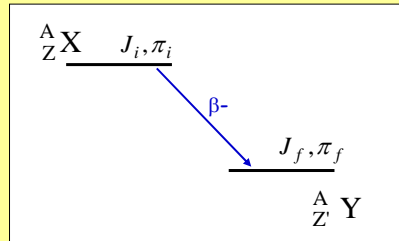


Figure 5-8: Systematics of observed $\log ft$ values. The grey area in the upper panel shows 718 cases of $0^+ \rightarrow 1^+$ allowed transitions, and the remaining 1741 cases of other allowed decays are shown by the white histogram. The peak of the distribution for the 24 cases of $0^+ \rightarrow 0^+$ superallowed decay is indicated by the arrow.

Nuclear case: $\frac{A}{Z}X \rightarrow \frac{A}{Z'}Y + e^- + \bar{\nu}_e$



Conservation laws:

$$\vec{J}_i = \vec{J}_f + \vec{S} + \vec{L}$$

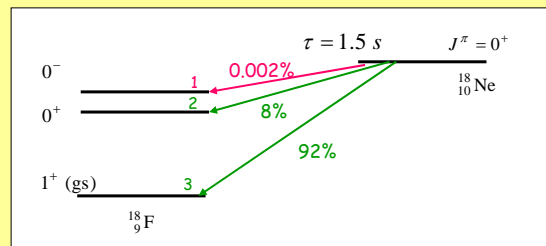
$$\pi_i = \pi_f (-1)^L$$

with $S = 0$ (Fermi) or $S = 1$ (Gamow-Teller)

Smallest value of L that is consistent with conservation laws will dominate the transition.

Example: β^+ decay of ^{18}Ne

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Branching ratio (BR): the fraction of decays that go to a particular final state.

In this case, $\lambda_{\text{total}} = 1/\tau = 0.667 \text{ sec}^{-1}$; $\lambda = \lambda_1 + \lambda_2 + \lambda_3$, with $\lambda_i = \text{BR}(i) \lambda_{\text{total}}$

Transition 1: $0^+ \rightarrow 0^-$ This is a **first forbidden GT decay**, with the slowest partial rate:

$$\vec{0} = \vec{0} + \vec{S} + \vec{L}; \quad (+) = (-) \times (-1)^L \rightarrow L=1, S=1$$

Transition 2: $0^+ \rightarrow 0^+$ This is an **allowed Fermi decay**:

$$\vec{0} = \vec{0} + \vec{S} + \vec{L}; \quad (+) = (+) \times (-1)^L \rightarrow L=0, S=0$$

Transition 3: $0^+ \rightarrow 1^+$ This is an **allowed Gamow-Teller decay**

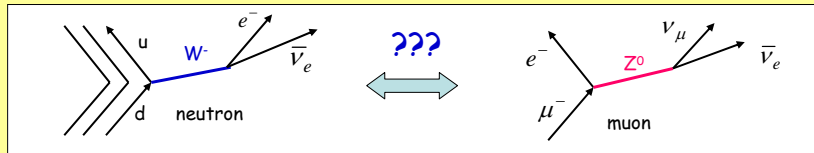
$$\vec{0} = \vec{1} + \vec{S} + \vec{L}; \quad (+) = (+) \times (-1)^L \rightarrow L=0, S=1$$

We have, so far two coupling constants for (nuclear) beta decay: G_V ($S=0$ decays) and G_A ($S=1$). These set the overall scale of the interaction, with G_V determined from the transition rates for "superallowed" $0^+ \rightarrow 0^+$ nuclear decays, and G_A from Gamow-Teller decays ($0^+ \rightarrow 1^+$ and vice versa).

Other related processes:

1. **muon decay:** $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ or $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

- a "purely leptonic" weak decay -- no quarks before or after!
- no change of electric charge; must be "mediated" by the neutral Z^0 boson
- no "Fermi function" needed, since no Coulomb effects in the final state.
- analogous to neutron decay, so we can try the same formalism, assuming weak interactions for quarks and leptons are the same



Muon lifetime implications:

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measured lifetime: $\tau = 2.19703 \pm 0.00004 \text{ } \mu\text{s}$

$$\mu^\pm \rightarrow e^\pm + \nu_e / \bar{\nu}_e + \bar{\nu}_\mu / \nu_\mu$$

theoretical prediction:

$$\tau = \frac{192 \pi^3 \hbar^7}{G^2 m_\mu^5 c^4}$$

(our prediction, integrated over phase space for the two neutrino types!)

Muon decay gives a weak coupling constant G that is about 2.5% larger than in nuclear beta decays....

or alternatively, the coupling constant for the $d \rightarrow u$ quark weak transition is about 2.5% smaller than that for the $\mu \rightarrow e$ lepton weak transition.

2. **Pion decay:** $\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$

Another $d \rightarrow u$ quark transition; rate is consistent with the same coupling constants as nuclear beta decay

3. **K meson decay** $K^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$ ($\bar{u}s \rightarrow \bar{u}u + e^- + \bar{\nu}_e$)

This is an $s \rightarrow u$ quark transition; rate is much smaller than the equivalent $d \rightarrow u$ rate; coupling constants are reduced to about 20% of nuclear beta decay values

- There are hundreds of examples of weak decays in nuclear and particle physics.
- **Purely leptonic rates** are all consistent with a **single weak coupling constant G**
- **Hadronic rates**, involving quark transitions, occur at a comparable scale but with **consistent differences that depend on the type of quarks involved**.
- A simple pattern emerges if we assume that **the quarks that participate in weak interactions are linear combinations of the strong interaction eigenstates**, represented by a **unitary matrix** called the CKM (Cabbibo-Kobayashi-Maskawa) matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \times \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

weak eigenstates Unitary matrix, like a rotation matrix - preserves "length" strong eigenstates

$d' = V_{ud} d + V_{us} s + V_{ub} b, \text{ etc...}$

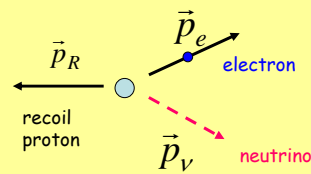
- Instead of a $d \rightarrow u$ transition in neutron beta decay, **only the contribution from the weak eigenstate d' plays a role**, and the weak coupling constant is effectively reduced by a factor $V_{ud} = 0.974$.
- Similarly, instead of an $s \rightarrow u$ transition in kaon decay, we have an $s' \rightarrow u$ transition, effectively reducing the weak coupling constant by a factor $V_{us} = 0.220$.
- Studies of a large number of particle decays and beta transitions have effectively "mapped out" the CKM matrix as follows: (Particle Data Group, 2004)

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 0.974 & 0.220 & 0.004 \\ 0.224 & 0.996 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{bmatrix}$$

0.9739 to 0.9751	0.221 to 0.227	0.0029 to 0.0045
0.221 to 0.227	0.9730 to 0.9744	0.039 to 0.044
0.0048 to 0.014	0.037 to 0.043	0.9990 to 0.9992

 2σ limits

- **Diagonal terms dominate the CKM matrix**
- All "large" terms are real; small imaginary component in lower right 2x2 submatrix allows for time reversal, or alternatively "CP violation" -- a hot research topic!

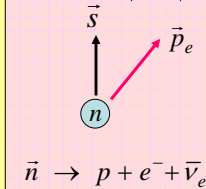


measuring spin-momentum correlations for the decay of **polarized** neutrons yields additional information (**neutron spin: \vec{s}**)
→ correlation coefficients: **a**, **A**, **B**:

$$A = -2 \frac{-G_A G_V + G_A^2}{G_V^2 + 3G_A^2}, \quad \tau = \frac{\text{constant}}{G_V^2 + 3G_A^2}$$

$$\lambda_{if} \propto p_e E_e (Q - E_e)^2 \left[1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \langle \vec{s} \rangle \cdot \left(A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} \right) \right] dE_e d\Omega_e d\Omega_\nu$$

β asymmetry **A**, the correlation $\langle \vec{s} \cdot \vec{p}_e \rangle$

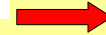


"little **a**" and "**B**" are hard to measure because one cannot determine the neutrino momentum directly.

The best additional measurement is the "**big A**" coefficient, which gives an independent constraint from the neutron lifetime, but **one has to control and measure the neutron spin direction and measure the electron momentum / energy very precisely...**

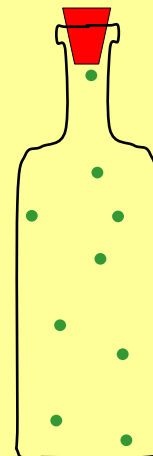
new experiment with ultra cold neutrons:

<http://www.krl.caltech.edu/ucn/>



What are Ultra Cold Neutrons (UCN) ?

- UCN are neutrons that are moving so slowly that they are totally reflected from a variety of materials.
- They can be confined in material bottles for long periods of time.
- Typical parameters:
 - velocity < 8 m/s
 - temperature < 4 mK
 - kinetic energy < 300 neV
- Interactions:
 - gravity: $V=mgh$
 - **weak interaction** (allows UCN to decay)
 - **magnetic fields**: $V=-\mu \cdot B$
(100 % polarization by passing through a magnet !)
 - strong interaction



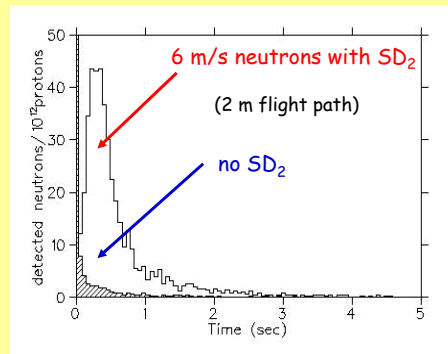
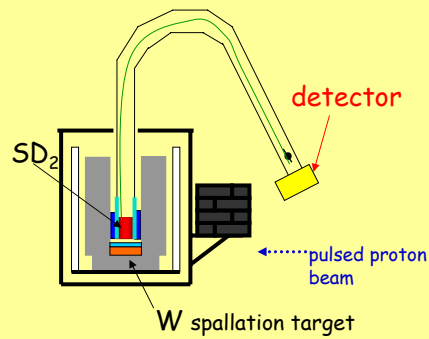
slides courtesy Prof. J. Martin, U. Wpg.

Cold n

UCN

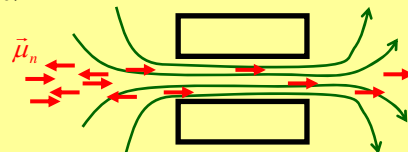
phonon

First UCN generation at Los Alamos:



UCNA Advantages: Polarization and Background

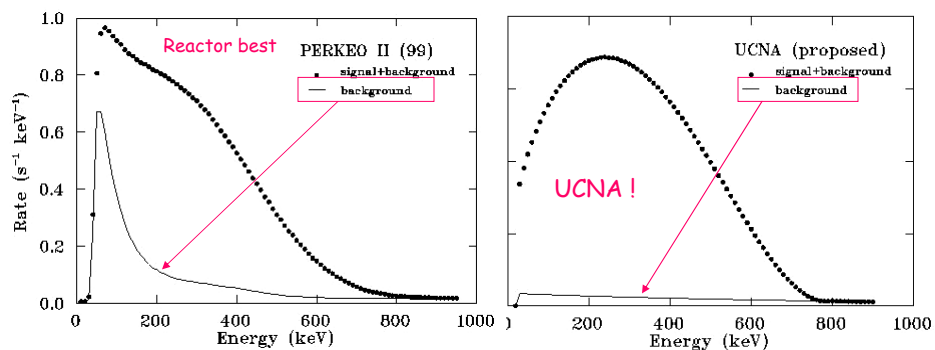
Ultra cold neutrons with the wrong spin direction can't make it through a large magnetic field!

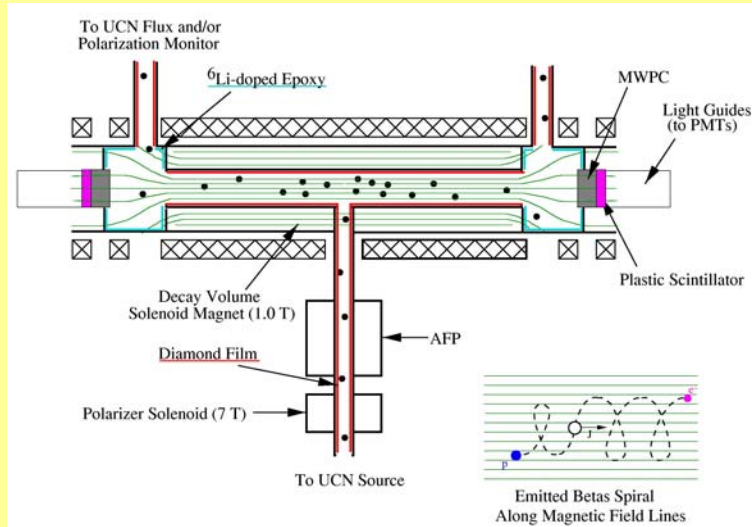


$B > 6 \text{ T}$
 $\rightarrow P = 100\%$

limitation - magnetic scattering from walls, etc.

Background reduction via pulsed source:



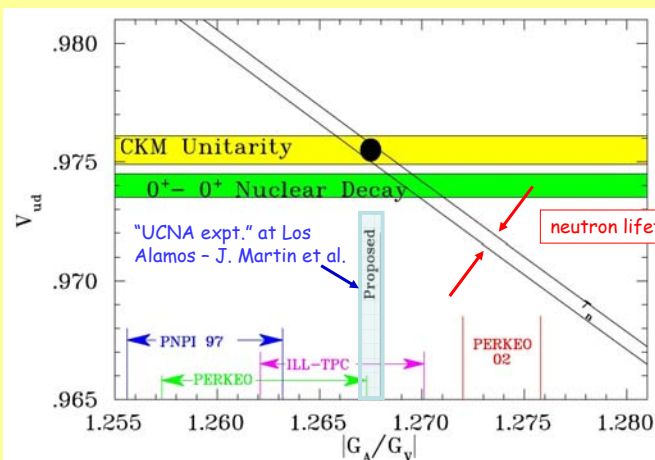


Issues: electron backscattering from detector surface (similar issue in lifetime expt.)
neutron depolarization by scattering from the walls ($\sim 0.1\%$)

$$V_{ij}^{-1} = V_{ij}^* \rightarrow V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \quad ? \quad (\text{the best-tested row of } V_{ij})$$

world data, 2004: $V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9967 \pm 0.0014$

2σ discrepancy:
contentious issue...



an additional generation of quarks is not entirely ruled out by the CKM unitarity test at the present time!

Leptons	ν_e	ν_μ	ν_τ	Z
	e	μ	τ	W
	u	c	t	γ
Quarks	d	s	b	g
	e	μ	τ	W

Force Carriers

horizontal scale - measurements of "A" in neutron decay